Lecture 6

- Cut off frequency of a waveguide

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon = A^2 + B^2$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon = \gamma^2$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} = \alpha + j\beta$$

At lower Frequencies

$$\omega^2\mu\varepsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

higher frequencies

$$\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

the frequency at which γ just become zero is defined as cut - off frequency

at $\mathrm{f} = \mathrm{f_c}, \gamma = 0$ or $\omega = 2\pi\mathrm{f} = 2\pi\mathrm{f_c} = \omega_c$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c^{\,2}\mu\varepsilon$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}$$

$$f_c = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}$$

$$f_c = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{\frac{1}{2}}$$

$$\lambda_c = \frac{c}{f} = c \Big/ \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{\frac{1}{2}}$$

$$\lambda_{cm,n} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$$

Guided wave length

- Guided wave length : distance travelled by a wave in order to under go a phase shift of 2 π radians
- Phase velocity: the rate at which the wave changes its phase in terms of the guided wavelength
- Group velocity: rate at which wave propagates through the guided wave length

$$\lambda_g = \frac{2\pi}{\beta} \quad \text{guided wave length}$$

$$V_p = \frac{\lambda_g}{unittime} = \lambda_g . f = \frac{2\pi}{\beta} f = \frac{\omega}{\beta}$$

$$V_g = \frac{d\omega}{d\beta}$$

Expression for phase velocity and group velocity

- Expression for phase velocity and group velocity

$$h^2 = \gamma^2 + \omega^2 \mu\varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\varepsilon$$

$$at\ \mathrm{f} = \mathrm{f}_c, \omega = \omega_c, \gamma = 0$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega_c{}^2 \mu\varepsilon$$

$$\gamma^2 = (j\beta)^2 = \omega_c{}^2 \mu\varepsilon - \omega^2 \mu\varepsilon$$

$$\gamma^2 = \beta^2 = \omega^2 \mu\varepsilon - \omega_c{}^2 \mu\varepsilon$$

$$\beta = \sqrt{\omega^2 \mu\varepsilon - \omega_c{}^2 \mu\varepsilon}$$

$$\beta = \sqrt{(\omega^2 - \omega_c{}^2)\mu\varepsilon}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{(\omega^2 - \omega_c{}^2)\mu\varepsilon}}$$

$$V_p = \frac{\omega}{\sqrt{(\omega^2 - \omega_c{}^2)\mu\varepsilon}}$$

$$V_p = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{\left(1-\frac{f_c}{f}\right)^2}.}$$

$$f_c = \frac{c}{\lambda_c} and f = \frac{c}{\lambda_o}$$

$$V_p = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}.} = \frac{c}{\sqrt{1-\left(\frac{\lambda_o}{\lambda c}\right)^2}.}$$

$$V_g = c\sqrt{1-\left(\frac{\lambda_o}{\lambda c}\right)^2}.$$

$$V_p V_g = c^2$$

$$\beta = \sqrt{(\omega^2 - \omega_c{}^2)\mu\varepsilon}$$

$$\frac{d\beta}{d\omega} = \frac{1}{2\sqrt{(\omega^2 - \omega_c{}^2)\mu\varepsilon}}.2\omega\mu\varepsilon$$

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$V_g.V_p = c^2$$

Relation between them

$$V_p = \lambda_g . f$$

$$V_p = \frac{\lambda_g}{\lambda_0} . c$$

$$V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda c} \right)^2}}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda c} \right)^2}}$$